Constraints on the anomalous $WW\gamma$ couplings from $b \to s\gamma$

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Abstract

We study contributions to $b \to s\gamma$ from anomalous $WW\gamma$ interactions. Although these anomalous interactions are not renormalizable, the contributions are cut-off independent. Using recent results from the CLEO collaboration on inclusive radiative B decays, we obtain bounds for the anomalous CP conserving and CP violating couplings. The constraints on the CP conserving couplings are comparable with or better than constraints from other experiments.

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The Minimal Standard Model (MSM) of electroweak interactions is in very good agreement with present experimental data. However, there are still many questions unanswered. The structure of the MSM has to be tested in fine detail. One particular interesting point is to find out the structure of the self-interaction of the electroweak bosons. Study of this will help us to establish whether the weak bosons are gauge particles with interactions predicted by the MSM, or gauge particles of some extensions of the MSM which predict different interactions at loop levels, or even non-gauge particles whose self-interactions at low energies are described by effective interactions. In general there will be more self-interaction terms than the tree level MSM terms - these additional terms are refered to as the anomalous couplings. It is important to find out experimentally what are the allowed regions for these anomalous couplings. In this paper we will study constraints on the anomalous $WW\gamma$ couplings using experimental data from $b \to s\gamma$.

The CLEO collabortion has recently observed [1] the exclusive decay $B \to K^*\gamma$ with a branching ratio $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ and placed an upper limit on the inclusive quark-level process $b \to s\gamma$ of $B(b \to s\gamma) < 5.4 \times 10^{-4}$ at 95% CL. In the MSM, $b \to s\gamma$ is induced at the one loop level [2,3]. The inclusive branching ratio is typically about 2×10^{-4}, which is consistent with the experiment. The theoretical prediction for the exclusive decay $B \to K^*\gamma$ depends on hadronic form factors which are not well determined at present. The ratio $Br(B \to K^*\gamma)/Br(b \to s\gamma)$ is estimated to be between 0.4 to 0.04 [3]. The MSM is not in conflict witht the experimental data. Due to the large uncertainty associated with the hadronic form factors for $B \to K^*\gamma$, in our study we will only consider constraints on the anomalous $WW\gamma$ couplings from the upper bound on $Br(b \to s\gamma)$.

The most general form, invariant under $U(1)_{em}$, for the anomalous $WW\gamma$ interactions can be parametrized as

$$L = i\kappa W_\mu^+ W_\nu^- F^{\mu\nu} + i\frac{\lambda}{M_W^2}W_{\sigma\rho}^+ W^{-\rho\delta}F_\delta^\sigma$$
$$+ i\tilde{\kappa}W_\mu^+ W_\nu^- \tilde{F}_{\mu\nu} + i\frac{\tilde{\lambda}}{M_W^2}W_{\sigma\rho}^+ W^{-\rho\delta}\tilde{F}_\delta^\sigma , \tag{1}$$

where $W_\mu^\pm$ are the W-boson fields, $W_{\mu\nu}$ and $F_{\mu\nu}$ are the W-boson and photon field strengths

respectively, and $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. The terms proportional to κ and λ are CP conserving and $\tilde{\kappa}$ and $\tilde{\lambda}$ are CP violating. The contributions to $b \to s\gamma$ from the κ and λ terms have been studied by Chia [4] and Peterson [5]. Here we will consider all the contributions and give up-dated constraints on κ and λ, and new constraints on $\tilde{\kappa}$ and $\tilde{\lambda}$.

The process $b \to s\gamma$ is induced at the one loop level. The effective Hamiltonian is given by

$$H_{eff} = i\frac{g^2}{2}\epsilon^\mu V_{tb}V_{ts}^* \bar{s}\gamma_\alpha\gamma_\nu\gamma_\beta\frac{1-\gamma_5}{2}b$$
$$\times \int \frac{dk^4}{(2\pi)^4} \frac{k^\nu(g^{\alpha\alpha'} - \frac{k^{+\alpha}k^{+\alpha'}}{M_W^2})(g^{\beta\beta'} - \frac{k^{-\beta}k^{-\beta'}}{M_W^2})\Gamma_{\mu\alpha'\beta'}(q, k^+, k^-)}{(k^2 - m_t^2)((p-k)^2 - M_W^2)((p'-k)^2 - M_W^2)} , \qquad (2)$$

where

$$\Gamma_{\mu\alpha\beta}(q, k^+, k^-) = -\kappa(g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha) - \tilde{\kappa}\epsilon_{\mu\alpha\beta\rho}q^\rho$$
$$+ \frac{\lambda}{M_W^2}(g_\alpha^\rho k^{+\delta} - g_\alpha^\delta k^{+\rho})(g_\beta^\delta k^{-\sigma} - g_\beta^\sigma k^{-\delta})(g_\rho^\mu q^\sigma - g_\mu^\sigma q^\rho)$$
$$+ \frac{\tilde{\lambda}}{M_W^2}(g_\alpha^\rho k^{+\delta} - g_\alpha^\delta k^{+\rho})(g_\beta^\delta k^{-\sigma} - g_\beta^\sigma k^{-\delta})\epsilon_{\rho\sigma\mu\tau}q^\tau ,$$

where k, p, and p' are the internal, b-quark and s-quark momentum respectively, $q = p' - p$, $k^+ = p - k$ and $k^- = k - p'$, and ϵ^μ is the photon polarization. Performing the standard Feynman parametrization, we have

$$H_{eff} = ig^2\epsilon^\mu V_{tb}V_{ts}^* \bar{s}\gamma_\alpha\gamma_\nu\gamma_\beta\frac{1-\gamma_5}{2}b$$
$$\times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{k^\nu(g^{\alpha\alpha'} - \frac{k^{+\alpha}k^{+\alpha'}}{M_W^2})(g^{\beta\beta'} - \frac{k^{-\beta}k^{-\beta'}}{M_W^2})\Gamma_{\mu\alpha'\beta'}(q, k^+, k^-)}{(k^2 - 2k\cdot(xp + yp') - (m_t^2 + (M_W^2 - m_t^2)(x+y))^3} .$$
$$\qquad (3)$$

Substituting $k' = k - (xp + yp')$ into eq.(3), we find only terms quadratic in k' contribute to $b \to s\gamma$. Higher order terms vanish and lower order terms are supressed by factors like $\frac{m_b^2}{M_W^2}$ or $\frac{m_s^2}{M_W^2}$, and can be safely neglected. We find the effective Hamiltonian H_{eff}^γ responsible for $b \to s\gamma$ to be

$$H_{eff}^\gamma = \frac{g^2}{2M_W^2}V_{tb}V_{ts}^* m_b \bar{s}\sigma_{\mu\nu}\epsilon^\mu q^\nu\frac{1+\gamma_5}{2}b$$

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$$\times \int_0^1 dx \int_0^{1-x} dy \int \frac{dk^4}{(2\pi)^4} \frac{(\kappa - i\tilde{\kappa})(2(x+y) - k'^2) + (\lambda - i\tilde{\lambda})k'^2(2 - 3(x+y))}{\beta_t + (1 - \beta_t)(x+y)}, \quad (4)$$

where $\beta_t = \frac{m_t^2}{M_W^2}$. In the above we have neglected terms proportional to m_s and have used the equation of motion and the identities: $\bar{s}\sigma_{\mu\nu}\gamma_5 b = -\frac{\epsilon_{\mu\nu\alpha\beta}}{2}\bar{s}\gamma^\alpha\gamma^\beta b$, and $\bar{s}\sigma_{\mu\nu}b = -\frac{\epsilon_{\mu\nu\alpha\beta}}{2}\bar{s}\gamma^\alpha\gamma^\beta\gamma_5 b$. After integrating over k', we obtain

$$H_{eff}^\gamma = i\frac{g^2}{32\pi^2 M_W^2}V_{tb}V_{ts}^* m_b \bar{s}\sigma_{\mu\nu}\epsilon^\mu q^\nu \frac{1 + \gamma_5}{2}b$$
$$\times [(\kappa - i\tilde{\kappa})(I_1(\beta_t) - I_1(0)) + (\lambda - i\tilde{\lambda})(I_2(\beta_t) - I_2(0))], \quad (5)$$

with

$$I_1(x) = \int_0^1 dx \int_0^{1-x} dy[\frac{x+y}{\beta_t + (1 - \beta_t)(x+y)} - \ln(\beta_t + (1 - \beta_t)(x+y))]$$
$$I_2(x) = \int_0^1 dx \int_0^{1-x} dy(2 - 3(x+y))\ln(\beta_t + (1 - \beta_t)(x+y)). \quad (6)$$

Here we have used the GIM mechanism to cancel out terms which do not depend on internal quark masses. The final expression for H_{eff}^γ is given by

$$H_{eff}^\gamma = iG_2 m_b \bar{s}\sigma_{\mu\nu}\epsilon^\mu q^\nu \frac{1 + \gamma_5}{2}b, \quad (7)$$

where

$$G_2 = \frac{G_F}{\sqrt{2}}\frac{e}{4\pi^2}V_{tb}V_{ts}^* G_A(\frac{m_t^2}{M_W^2}), \quad (8)$$

with $G_A(x)$ given by

$$G_A(x) = (\frac{\kappa}{e} - i\frac{\tilde{\kappa}}{e})(\frac{x}{(1-x)^2} + \frac{x^2(3-x)lnx}{2(1-x)^3})$$
$$-(\frac{\lambda}{e} - i\frac{\tilde{\lambda}}{e})(\frac{x(1+x)}{2(1-x)^2} + \frac{x^2 lnx}{(1-x)^3}). \quad (9)$$

We emphasise that the result is finite and that we do not need the use of cut-offs to analyze our results.

Combining the contribution from the MSM and using the leading QCD correction obtained by Grintein, Springer and Wise in Ref. [2], we obtain the total contribution to $b \to s\gamma$ of the form of eq.(7), but with G_2 replaced by G_2^t. Here G_2^t is given by

$$G_2^t = i\frac{G_F}{\sqrt{2}}\frac{e}{4\pi^2}V_{tb}V_{ts}^*C_7(\frac{m_t^2}{M_W^2}) \; ,$$

$$C_7(x) = \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{16/23}\left(C_7'(x) + G_A(x) - \frac{8}{3}C_8'(x)\left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{2/23}\right]\right.$$
$$\left. +\frac{464}{513}\left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{19/23}\right]\right) \; , \tag{10}$$

and

$$C_7'(x) = \frac{x}{(1-x)^3}(\frac{2}{3}x^2 + \frac{5}{12}x - \frac{7}{12} + \frac{(\frac{3}{2}x^2 - x)lnx}{1-x})$$
$$C_8'(x) = \frac{x}{2(1-x)^3}(\frac{1}{2}x^2 - \frac{5}{2}x - 1 - \frac{3xlnx}{1-x}) \; . \tag{11}$$

The decay width for $b \rightarrow s\gamma$ is given by

$$\Gamma(b \rightarrow s\gamma) = \frac{|G_2|^2 m_b^5}{16\pi} \; . \tag{12}$$

To obtain $Br(b \rightarrow s\gamma)$, we use the latest data on the semileptonic branching ratio [6] $Br(b \rightarrow X_c e\nu) = 0.108$ to scale the inclusive $b \rightarrow s\gamma$ rate. This proceedue removes the uncertainties associated with $(m_b)^5$ and KM factors. We have

$$Br(b \rightarrow s\gamma) = \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2}Br(b \rightarrow X_c e\nu)\frac{3\alpha_{em}}{2\pi\rho\eta}|C_7(\frac{m_t^2}{M_W^2})|^2 \; , \tag{13}$$

where the phase space factor $\rho = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln r$ with $r = m_c/m_b$ and the QCD correction factor $\eta = 1 - 2f(r,0,0)\alpha_s(m_b)/3\pi$ with $f(r,0,0) = 2.41$ [7]. In our numerical analysis, we will use [8]: $\alpha_s(M_W) = 0.105$, $m_b = 5GeV$, $m_c = 1.5GeV$, $|V_{tb}V_{ts}^*|/|V_{cb}| \approx 1$ and let the top quark mass m_t and the anomalous couplings vary.

The contributions from the κ and λ terms can either increase or decrease the MSM prediction for $Br(b \rightarrow s\gamma)$ depending on the ranges and the signs of the parameters. If we set the signs of κ and λ to be positive, the contribution from the λ term has the same sign as that of the MSM, while the contribution from the κ term has opposite sign. It is confusing to do the complete analysis if we keep κ, $\tilde{\kappa}$, λ, and $\tilde{\lambda}$ arbitrary. For simplicity we will consider constraints on individual anomalous coupling; that is, we let only one anomalous coupling

to be non-zero when carrying out the analysis. The resulting bounds on the anomalous couplings at the are given in Table 1. The constraint on κ is the tightest, for example, for $m_t = 150 GeV$, κ/e is constrained to be between 2.5 and -0.44. This bound is better than that obtained from the muon anomalous magnetic dipole moment [9]. Our bound restrict κ to a range than that derived from an analysis of the experimental data at CDF, which is $3.7 > \kappa/e > -2.6$ at 96% [10], and if κ is negative this constraint is better. The constraint on λ is weaker, for $m_t = 150 GeV$, λ/e is restricted to be between -7.2 and 1.3. This bound is comparable with the constraint from the muon anomalous magnetic dipole moment. All constraints become tighter when m_t is increased. It is interesting to note that if the branching ratio of $b \rightarrow s\gamma$ turns out to be significantly smaller than 2×10^{-4}, it may be an indication for non-zero anomalous $WW\gamma$ coupling. Complete cancellation between the MSM and the anomalous contributions can occur for certain values of κ and λ. In Tables 2.and 3., we give $Br(b \rightarrow s\gamma)$ as functions of κ and λ for $m_t = 150 GeV$. From Tables 2. and 3., we see that if a lower bound for $Br(b \rightarrow s\gamma)$ will be well established, one can exclude some regions in the allowed parameter space given in Table 1.

Due to phase differences in the amplitudes, the contributions from the $\tilde{\kappa}$ and $\tilde{\lambda}$ terms do not interfere with the MSM contribution and thus appear only quadratically in the modified expression for $Br(b \rightarrow s\gamma)$. Therefore non-zero $\tilde{\kappa}$ and $\tilde{\lambda}$ can only increase the MSM prediction for $Br(b \rightarrow s\gamma)$. However cancellation can occur between the contributions from the $\tilde{\kappa}$ and $\tilde{\lambda}$ terms. Again we will analyze the constraints on the couplings by keeping only one non-zero. We find that the typical constraints on $\tilde{\kappa}/e$ and $\tilde{\lambda}/e$ are of order one. The results are shown in Table 1. These bounds are weaker than the ones obtained from the experimental upper bound on the neutron and electron electric dipole moment [11]; however these bounds are cut-off dependent, and the present bounds are not.

The above analysis has neglected a number of theoretical uncertainties. There are uncertainties in the numerical factors for the QCD corrections [12]. However these corrections are small, at most a few percent. There is also uncertainty from the KM matrix elements [8], but this uncertainty is again small. The biggest uncertainty is in the phase space factor

ρ. Within the allowed values for m_c/m_b, ρ can vary between 0.41 to 0.65. Our choice of $m_c/m_b = 0.3$ correcponds to $\rho = 0.52$. Had we used a smaller value for ρ, we would have obtained stronger constraints. The uncertainty due to possible variations in ρ can have a factor of two effect on the constraints of the anomalous couplings.

It is possible to carry out an analysis keeping all anomalous couplings arbitrary. This exercise will obtain a correlated allowed range in the parameter space. However we do not think this will provide substantial new information. In fact if one chooses values for κ and λ such that their contributions to $b \rightarrow s\gamma$ cancel out, any value for κ is allowed. Similar situation happens to $\tilde{\kappa}$ and $\tilde{\lambda}$.

In summary, we have obtained constraints on the anomalous $WW\gamma$ couplings by using the recent results from CLEO collaboration on the inclusive radiative B decays. The constraints on the CP violating couplings are weaker than that obtained from the neutron and electron electric dipole moment. For CP conserving couplings, the constraints obtained in this paper are comparable with or better than other constraints. Improved experimental data on $b \rightarrow s\gamma$, especially well established lower bound on the branching ratio can further constrain the anomalous couplings.

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Note added.- After the completion of this paperr we became aware of a paper by T. Rizzo [13] in which a similar analysis for κ and λ has been carried out. In addition to the analysis in our paper, in Ref. [13] information obtained from hadron colliders are also used, and the correlated allowed regions for κ and λ are presented. Our results agree with that obtained by Rizzo. The results on $\tilde{\kappa}$ and $\tilde{\lambda}$ are, however, new. Note that there is a sign difference in the definition of the parameter κ.

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TABLE I. The constraints for the anomalous $WW\gamma$ couplings.

m_t(GeV)	100	125	150	175	200
κ/e	$3.54 \sim -0.86$	$2.88 \sim -0.60$	$2.48 \sim -0.44$	$2.2 \sim -0.34$	$2.0 \sim -0.28$
λ/e	$-9.28 \sim 2.28$	$-7.98 \sim 1.68$	$-7.2 \sim 1.32$	$-6.7 \sim 1.08$	$-6.34 \sim 0.9$
$|\tilde{\kappa}|/e$	<1.76	<1.32	<1.06	<0.88	<0.76
$|\tilde{\lambda}|/e$	<4.60	<3.66	<3.08	<2.68	<2.4

TABLE II. $Br(b \to s\gamma)$ vs. κ for $m_t = 150 GeV$.

κ/e	3.0	2.5	2.0	1.5	0.5	0.0	-0.5	-1.0
$Br \times 10^4$	9.78	5.47	2.4	0.58	7×10^{-4}	2.57	5.72	10

TABLE III. $Br(b \to s\gamma)$ vs. λ for $m_t = 150 GeV$.

λ/e	1.5	0.0	-1.5	-3.0	-4.5	-6.0	-7.5
$Br \times 10^4$	5.58	2.57	0.62	8×10^{-4}	0.71	2.76	6.13